THE HALL EFFECT IN AN ELECTRICALLY CONDUCTING FLUID FOR VARIABLE MAGNETIC FIELD AND HIGH MAGNETIC REYNOLDS NUMBERS

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Hall phenomena in an electrically conducting fluid with a variable magnetic field were considered in [1]. In that paper the basic characteristics of the above-mentioned phenomena are determined, with certain unimportant constraints, for the case of fluid motion along a channel of rectangular cross section in a traveling magnetic field. The magnetic Reynolds number was assumed to be small, and a solution was given for the induction field in the form of a series in powers of the indicated parameter. Quantitative estimates based on the data of [1] are impossible in the case of relatively high electrical conductivity of the fluid, although certain conclusions of a qualitative nature remain valid. There is thus reason to consider the case of high magnetic Reynolds numbers. This will also allow a fuller picture of the characteristic Hall effect phenomena to be constructed for a variable magnetic field.

We make the following assumptions: the fluid flows along a channel of rectangular cross section, the channel walls are nonconducting, the fluid motion is constant, ion slip does not occur, the external magnetic field is created by windings located on the inner surfaces of an inductor, i.e., on the z = 0 and z = ∂ planes (∂ is the channel height), the linear load of each of the windings has one component in the form of a traveling wave.

We shall determine the induction field.

We isolate an infinitely thin layer of fluid along the channel axis z = ∂/2. Assuming

$$h_i = \varphi_i(z) f_i(t, x, y),$$

$$\varphi_i(z) = \alpha_{i1}(z - {}^1/_2\,\delta) + \alpha_{i3}(z - {}^1/_2\,\delta)^3 + \ldots \quad (i = x, y).$$

We can write the equation for the Z component of the induction field for this layer as

$$L(h) = \Delta h - \beta_m \cos(t - x)\frac{\partial h}{\partial y} - \varepsilon\left[\frac{\partial h}{\partial t} + (1 - s)\frac{\partial h}{\partial x}\right] -$$

$$- \varepsilon s \cos(t - x) = 0. \qquad (1)$$

Here Δ is the Laplace operator, h is the induction field strength, β_m is the Hall parameter for induction, equal to the amplitude of the traveling wave field, s is the slip, ε the analog of the magnetic Reynolds number.

Then equation (1) is considered in the region

$$(0 < t < l) \times \Omega\,(-p\pi < x < p\pi,\ 0 < y < d)$$

under the conditions

$$h_{(y=0)} = h_{(y=d)} = 0, \quad h_{(x=p\pi)} = h_{(x=-p\pi)}$$

$$\frac{\partial h}{\partial x_{(x=p\pi)}} = \frac{\partial h}{\partial x_{(x=-p\pi)}}, \quad h_{(t=0)} = 0.$$

Thus transient processes in the inductor circuits are not taken into account and it is assumed that the traveling wave is formed at the instant the inductor contact is made.

To solve equation (1), we make use of a method similar to that of Galerkin, as modified by M. I. Vishik for application to mixed boundary value problems with time [2]. We set

$$h^\circ = \frac{1}{2}\sum_1^m a_{m0}(t)\sin\frac{m\pi y}{d} +$$

$$+ \sum_1^{m,\,n}\left[a_{mn}(t)\cos nx + b_{mn}(t)\sin nx\right]\sin\frac{m\pi y}{d}. \qquad (2)$$

The functions

$$\sin(m\pi y/d)\cos nx, \qquad \sin(m\pi y/d),\ \sin nx$$

$$(m = 1, 2, \ldots;\ n = 0, 1, 2, \ldots)$$

form a complete linearly independent system of elements in the corresponding Hilbert space. Consequently, for m → ∞; n → ∞, h° should coincide with the solution of equation (1).

In (2) the quantities $a_{mn}(t)$, $b_{mn}(t)$ are unknown. To obtain the corresponding equations we carry out the scalar multiplication of L(h°) by the coordinate functions in the region Ω. We have

$$\frac{d}{dt}a_{mn}(t) + \frac{z_{mn}}{\varepsilon}a_{mn}(t) + (1 - s)\,nb_{mn}(t) +$$

$$+ \frac{\beta}{\varepsilon}\sum^k \frac{km}{k^2 - m^2}\left[(-1)^{k+m} - 1\right]\{[a_{k(n-1)}(t) + a_{k(n+1)}(t)]\cos t -$$

$$- [b_{k(n-1)}(t) - b_{k(n+1)}(t)]\sin t\} = \frac{2\delta_{mn}s}{m\pi}\left[(-1)^m - 1\right]\cos t$$

$$(m = 1, 2, \ldots;\ n = 0, 1, 2, \ldots), \qquad (3)$$

$$\frac{d}{dt}b_{mn}(t) + \frac{z_{mn}}{\varepsilon}b_{mn}(t) - (1 - s)\,na_{mn}(t) +$$

$$+ \frac{\beta}{\varepsilon}\sum^k \frac{km}{k^2 - m^2}\left[(-1)^{k+m} - 1\right]\{[b_{k(n-1)}(t) + b_{k(n+1)}(t)]\cos t +$$

$$+ [a_{k(n-1)}(t) - a_{k(n+1)}(t)]\sin t\} = \frac{2\delta_{mn}s}{m\pi}\left[(-1)^m - 1\right]\sin t$$

$$(m = 1, 2, \ldots;\ n = 0, 1, 2, \ldots), \qquad (4)$$

for initial conditions

$$a_{mn}(0) = 0, \qquad b_{mn}(0) = 0.$$

Here

$$z_{mn} = (m\pi/d)^2 + n^2, \quad \beta = \beta_m/d, \quad \delta_{m1} = 1, \quad \delta_{m(n\neq 1)} = 0.$$

Further, settling processes are not considered. In this case, in (3) and (4) we can discard all equations in which the sum of the indices associated with $a(t)$ and $b(t)$ is an odd number.

Equations (3) and (4) were solved numerically on a digital computer. The table gives values (rounded) of the variables at a series of points for the case $\varepsilon = 4.9$, $\beta = 5.57$, $s = -1$. In the extreme right-hand column the approximate expression for the variables is given.

The fact that $a_{mn}(t)$ and $b_{mn}(t)$ have an explicitly expressed sinusoidal character allows the procedure for determining the induction field to be simplified. If strict requirements of accuracy are not imposed, we may immediately set

$$a_{m0}(t) = \text{const}, \qquad a_{mn}(t) = a_{mn}\sin(nt - \psi_{mn}),$$
$$b_{mn}(t) = -b_{mn}\cos(nt - \psi_{mn}) \qquad (5)$$

in (2)

In this case, we can obtain algebraic equations, instead of a system of ordinary differential equations with periodic coefficients, by carrying out a scalar multiplication of the operator $L(h^\circ)$ by the coordinate functions in the region $\Omega \times (0 < t < 2\pi)$. We have

$$z_{mn}f_{mn} + \beta\sum^{k}\frac{km}{k^2 - m^2}[(-1)^{k+m} - 1][f_{k(n-1)} + f_{k(n+1)}] -$$
$$- \varepsilon sng_{mn} = -\frac{2\delta_{mn}\varepsilon s}{m\pi}[(-1)^m - 1], \qquad (6)$$

$$z_{mn}g_{mn} + \beta\sum^{k}\frac{km}{k^2 - m^2}[(-1)^{k+m} - 1]\cdot$$
$$\cdot[g_{k(n-1)} + g_{k(n+1)}] + \varepsilon snf_{mn} = 0 \qquad (7)$$

$$(f_{mn} = a_{mn}\sin\psi_{mn}, \quad g_{mn} = a_{mn}\cos\psi_{mn}, \quad f_{m0} = -a_{m0}).$$

Here as in (3) and (4) the sums of the indices of f and g are even numbers.

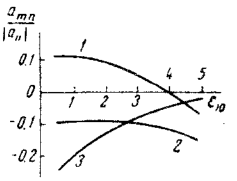

Fig. 1

Hence, for the case mentioned above, we obtain

$$a_{11} = -0.374, \quad a_{31} = -0.034, \quad a_{20} = -0.0894, \quad a_{22} = 0.0435,$$
$$\psi_{11} = 71.5°, \quad \psi_{31} = 83°, \quad \psi_{22} = 59°.$$

Thus both methods give good agreement for a_{mn} and poor agreement for ψ_{mn}. This is confirmed by data for other values of ε, β and s, which are not given here. The marked divergence of the strength of the double frequency field should not have much effect, since this field is small in comparison with the basic field (a_{11}).

The influence of the magnetic Reynolds number on the distribution of the induction field may easily be estimated from equations (6) and (7).

As the magnetic Reynolds number increases, the zero and double frequency field decreases; on the other hand, the basic frequency field (a_{31}) increases. Figure 1 gives the amplitude values of the fields referred to for $\beta = 5.57$ and $s = -1$ (curves 1, 2, and 3 correspond to a_{20}, a_{31} and a_{22}).

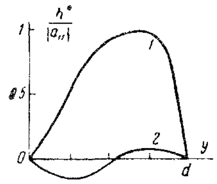

Fig. 2

The magnetic Reynolds number exerts a marked effect on the field distribution in the fluid.

The field distribution is less symmetric for low values of the Reynolds number, and, in this sense, the Hall effect is stronger (Fig. 2; $\varepsilon = 0.49$, $\beta = 5.57$, $s = -1$, $|a_{11}| = 0.033$, curve 1 corresponds to $t - x = 0$, curve 2 corresponds to the value $t - x = \pi/2$.

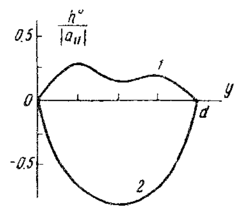

Fig. 3

The field distribution becomes more symmetric at high values of the magnetic Reynolds number, since the zero and double frequency components are less, but the corresponding curves become more flattened as a result of the increased basic frequency component (a_{31}) (Fig. 3: $\varepsilon = 49$, $\beta = 5.57$, $s = -1$, $|a| = 1.23$, curve 1 corresponds to $t - x = 0$, curve 2 to $t - x = \pi/2$). In the first case (Fig. 2) the field strength for $t - x = 0$ is considerably greater than for $t - x = \pi/2$; in the second case, the opposite state of affairs is observed. This is

t rad	186.6	188.2	189.7	191.3	192.9	
a_{11}	−0.0052	0.372	0.0206	−0.3719	0.001	$-0.372\sin(t - 75°)$
b_{11}	−0.371	0.0118	0.371	0.0037	−0.371	$0.371\cos(t - 75°)$
a_{31}	−0.006	0.0336	0.0074	−0.0339	−0.0054	$-0.0335\sin(t - 85.5°)$
b_{31}	−0.0348	−0.0065	0.0344	0.0079	−0.0348	$0.0345\cos(t - 85.5°)$
a_{20}	−0.0884	−0.0879	−0.0884	−0.0878	−0.0883	-0.0883
a_{22}	0.0579	−0.0438	0.0571	−0.0432	0.0582	$0.007 + 0.05\sin(2t - 68°)$
b_{22}	−0.004	0.0079	−0.0082	0.0124	−0.0023	$0.003 - 0.05\cos(2t - 68°)$

basically connected with the substantial difference of time phases φ_{mn} as a consequence of the different q of the medium in the cases mentioned.

It was assumed above that the amplitude of the external field is constant. Consequently, it is also assumed that the inductor is considerably wider than the channel. However, cases where these dimensions are either the same, or differ only insignificantly, are more realisitc. Here the external field does not remain constant over the channel width, and the distribution curve for this field is a symmetric function with respect to the channel axis.

It can easily be shown that it is possible to go over to equations (6) and (7) in this case also. Setting

$$\varphi(y) = \sum_{0}^{i} a_{2i}(2y - d)^{2i},$$

where $\varphi(y)$ is the amplitude of the external field, a_{2i} are coefficients depending on the inductor design and the type of exciting winding, we obtain

$$\int_{0}^{d} \varphi(y) \sin \frac{m\pi y}{d} \, dy = N\left[(-1)^{m} - 1\right].$$

Thus the factor N must be introduced in the right-hand side of equations (3) and (4). Since this factor is bounded, one can expect conditions (5) to be satisfied, if only approximately. This means that equations (6) and (7) can be applied even when the channel and inductor are of the same width, provided we set $\delta m_1 = N$.

REFERENCES

1. K. I. Kim, "Electromagnetic processes in an anisotropic electrically conducting fluid with a variable magnetic field," PMTF, no. 6, 1964.

2. M. I. Vishik, "Mixed boundary value problems and an approximate method of solving them," DAN SSSR, vol. 99, no. 2, 1954.

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